In category theory objects have no structure, everything we can say about objects comes through the relationship with other objects: morphisms.

> it is as if morphisms connects to hidden features inside objects. They reveal the inner structure of objects.

All the information about an object is encoded into its hom-functors.

More precisely

1. take any object $c \in \mathcal{C}$
2. we can use any other object $c' \in \mathcal{C}$ as a probe or as a vantage point and consider morphisms $f : c \to c'$
3. varying c' we get its covariant hom-functor $\mathcal{C}(c, -)$
4. and we say that $\mathcal{C}(c, -)$ or any other naturally isomorphic functor represents c
5. thus we get the Yoneda embedding of $\mathcal{C}$ into $[\mathcal{C}^{op}, \mathcal{S}et]$
6. using Yoneda lemma we can prove that Yoneda embeddings are **full and faithful** thus
   - all the information about $c \in \mathcal{C}$ is encoded into $\mathcal{C}(c, -)$
   - $a \cong b$ iff $\mathcal{C}(a, -) \cong \mathcal{C}(b, -)$
   - every natural transformation between hom-functors arise from a morphism between it represented objects
7. when an object is specified uniquely by its relationship with other objects it is said to satisfy a universal property

Similarly for the contravariant hom-functor $\mathcal{C}(-, c)$.

Going higher up on the abstraction ladder the important thing about functors is how they operate on morphisms: they preserve structure by preserving composition of morphisms.

But functors form a category by themselves so they become structureless objects in this category. Then the information about functors is encoded in the morphisms between them that is natural transformations.

See also

- The Yoneda Perspective