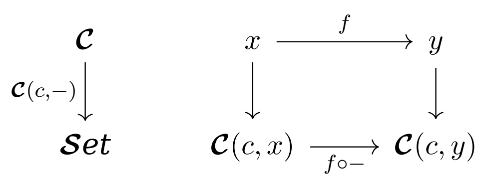
How is information about an object $c \in \mathcal{C}$ encoded in morphisms emanating from c (or inpinging into c)?

1. consider the set $\mathcal{C}(c, x)$ of morphisms emanating from c we keep c fixed and vary x so we want a *function* $\mathcal{C}(c, -)$
2. we are really interested in how the sets $\mathcal{C}(c, -)$ transform under a morphism $f : x \to y$ ($\mathcal{C}(c, x) \to \mathcal{C}(c, y)$) so we need to consider also $\mathcal{C}(c, f)$
3. in the end we need a functor

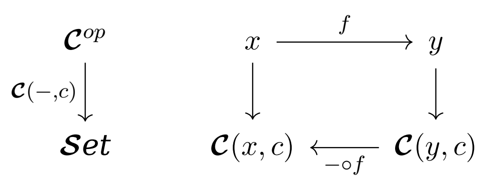
(similarly for impinging morphisms $\mathcal{C}(-, c)$)

Definition 0.1. Given a locally small category $\mathcal{C}$ for any object $c \in \mathcal{C}$ the covariant and contravariant **hom-functors**[1] are defined as

- $\mathcal{C}(c, -)$ maps any object $x \in \mathcal{C}$ to the hom-set $\mathcal{C}(c, x)$
- $\mathcal{C}(c, -)$ maps any morphism $f : x \to y$ by *pre-composition* $\mathcal{C}(c, f)g = f \circ g$



- $\mathcal{C}(-, c)$ maps any object $x \in \mathcal{C}$ to the hom-set $\mathcal{C}(x, c)$
- $\mathcal{C}(-, c)$ maps any morphism $f : x \to y$ by *post-composition* $\mathcal{C}(f, c)g = g \circ f$



[1] they are also known as **functors represented by c**

Note that hom-functors are uniquely determined by c.

See [1] Ch.1.1, [2] Ch.2.1.1.

[1] E. Riehl, *Category Theory in Context* (Dover, 2015).
[2] P. Perrone, *Notes on Category Theory with Examples from Basic Mathematics*, (2019).