Definition 0.1. Given a group G consider the functor

$$F : \mathcal{B}G \to \mathcal{C}$$

with

- $F\bullet = X$ where X is a fixed object $X \in \mathcal{C}$
- for any $f \in G$ an endomorphism $f_\star : X \to X$ ($Ff = f_\star$)

such that

- $h_\star g_\star = (hg)_\star$ for all $h, g \in G$
- $e_\star = \mathbf{1}_X$ where $e \in G$ is the group identity

thus F defines a **left action**[1] of the group G on the object $X \in \mathcal{C}$. Also

when $\mathcal{C}$ is	X is called
$\mathcal{S}et$	G-**set**
$\mathcal{V}ect_{\mathbb{K}}$	G-**representation**
$\mathcal{T}op$	G-**space**

Corollary 0.1. *When a group G acts functorially on an object $X \in \mathcal{C}$ its elements g act by **automorphism** $g_\star : X \to X$ (moreover $(g_\star)^{-1} = (g_\star^{-1})$).*

Proof. Any element $g \in G$ regarded as a morphism of $\mathcal{B}G$ is an *isomorphism* thus functoriality guarantees that $g_\star$ is an isomorphism as well. So for example for the action $\mathcal{B}G \to \mathcal{V}ect_{\mathbb{K}}$ on a vector space the linear map $g_\star : V \to V$ must be an *automorphism* of the vector space V.
□

See [1] Ch.1.3.

[1] E. Riehl, *Category Theory in Context* (Dover, 2015).

[1] a **right action** is a functor $\mathcal{B}G^{op} \to \mathcal{C}$ and the requirement for the endomorphism is $g^\star h^\star = (hg)^\star$