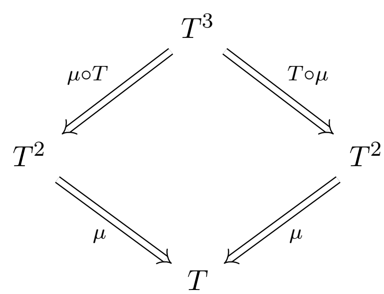
a monad is just a monoid in the category of endo-functors

Definition 0.1. A **monad** on a category $\mathcal{C}$ is a triple (T, η, μ)

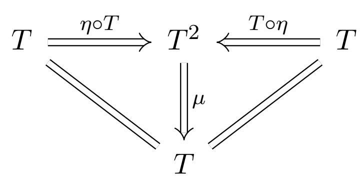
- $T : \mathcal{C} \to \mathcal{C}$ an endo-functor
- $\eta : \mathbf{1}_{\mathcal{C}} \Rightarrow T$ the **unit** natural transformation[1]
- $\mu : T^2 \Rightarrow T$ the **multiplication** natural transformation[2]

such that

1. the *associativity* diagram



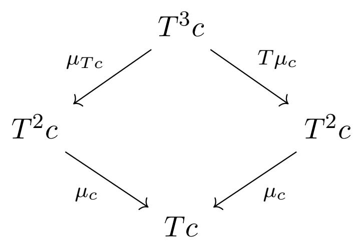
2. the *left and right unitality* diagrams

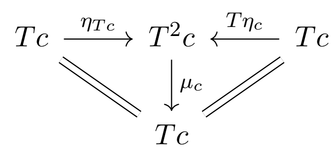


commute.

Taking the components at $c \in \mathcal{C}$ and making explicit the horizontal composition

[1] in Haskell `return`
[2] in Haskell `join`





See [1] Ch.5 and also

- Category Theory 3.2: Kleisli category - YouTube
- Category Theory 10.1: Monads - YouTube
- monad in nLab
- Monads (examples)

[1] P. Perrone, *Notes on Category Theory with Examples from Basic Mathematics*, (2019).