Consider the functor category $[\mathcal{C}, \mathcal{S}et]$, hom-functors $\mathcal{C}(c,-)$ are objects in $[\mathcal{C}, \mathcal{S}et]$ (for every $c \in \mathcal{C}$) and there are many other functors *naturally isomorphic to them*, these are called **representable** that is they are represented by *one* objects in $\mathcal{C}$ that uniquely determines them.

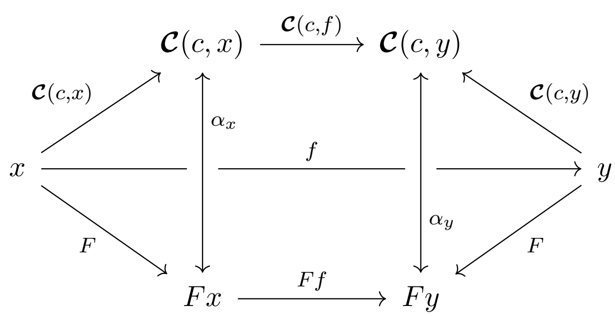
Definition 0.1. Given a locally small[1] category $\mathcal{C}$ a covariant (contravariant) functor $F : \mathcal{C} \to \mathcal{S}et$ is **representable** iff there exist

- an object $c \in \mathcal{C}$
- a natural isomorphism $F \cong \mathcal{C}(c,-)$ ($F \cong \mathcal{C}(-,c)$)

and we say that

- F is **represented by c**
- $c \in \mathcal{C}$ and $F \cong \mathcal{C}(c,-)$ ($F \cong \mathcal{C}(-,c)$) are a **representation** of F

The natural isomorphism $F \cong \mathcal{C}(c,-)$ is



See [1] Ch.2.1, [2] Ch.2.1.2, [3] Ch.14. Here are some examples.

See also

- Category Theory II 4.1: Representable Functors - YouTube

[1] in this way the hom-functors are valued in $\mathcal{S}et$

[1] E. Riehl, *Category Theory in Context* (Dover, 2015).
[2] P. Perrone, *Notes on Category Theory with Examples from Basic Mathematics*, (2019).
[3] B. Milewski, *Category Theory for Programmers* (2019).